Final Report

on

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Large Scale Disturbances Mechanism of Jet Noise Generation

Period covered by this report:

December 1, 1975 to June 30, 1976

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Introduction

This report covers the period from December 1, 1975 to June 30, 1976. During these seven months the resources of this grant (NSG-1021) were used to support a graduate student working on his doctoral dissertation entitled "Numerical Experiments on a Mechanism (Large Scale Disturbances) of Jet Noise Generation." The following is a description of the progress made in this respect. A summary of the distribution of financial support and publications pertaining to this grant is listed at the end of this report.

Large Scale Disturbances and Subsonic Jet Noise Generation

(1) Objectives of the investigation

The objectives of this study are:

(a) To determine through numerical experiments whether there are any orderly large scale structures in a high Reynolds number jet flow.

(b) In case orderly large scale structures do exist, to find out if they are responsible for the generation of the dominant part of jet noise.

(c) To examine the effect of upstream disturbances on jet noise generation.

In order to achieve these goals numerical simulation of the jet flow and the associated noise are to be carried out at the FSU computer center. The first step in performing the simulation is to develop a stable and efficient computer program for solving the appropriate compressible flow equations. This is done as follows.

(2) The governing equation

The governing equations, namely, continuity, momentum and energy equations are recasted into a compact vector form with respect to a cylindrical coordinate system (axisymmetry is assumed).

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial r} + \frac{\partial G}{\partial z} = H$$

where

$$U = (d,m,n,E)$$

$$H = (0,p,0,0)$$

$$F = \begin{pmatrix} m \\ \frac{3-\gamma}{2}\frac{m^2}{d} + (\gamma-1)(E-\frac{n^2}{2d}) \\ \frac{mn}{d} - (\frac{1}{Re} + KdU_c b)\frac{\partial}{\partial \gamma}(\frac{n}{d}) \\ \frac{\gamma m E}{d} + \frac{1-\gamma}{2}\frac{n(n^2+m^2)}{d^2} \end{pmatrix}$$

$$G = \begin{pmatrix} n \\ mn/d \\ \frac{3-\gamma}{2}\frac{n^2}{d} + (\gamma-1)(E-\frac{m^2}{2d}) \\ \frac{\gamma n E}{d} + \frac{1-\gamma}{2}\frac{n(n^2+m^2)}{d^2} \end{pmatrix}$$

$$d = \gamma\rho \ , \qquad m = \gamma\rho v \ , \qquad n = \gamma\rho u$$

$$E = \gamma\rho\ (\ell + \frac{u^2+v^2}{2})$$

ρ = density of fluid

(u,v) = velocity components in the (z,r) directions

U_c = center line velocity

b = half width of jet

$$K = \begin{cases} 0.04 \quad \tanh \quad 2(b-1.1) \quad , & \text{if } \dfrac{\partial \overline{u}}{\partial r} > 0.0001 \\ 0 \quad , & \text{if } \dfrac{\partial \overline{u}}{\partial r} < 0.0001 \end{cases}$$

In the above equations the effects of fine scale turbulence on large scale fluid motion are accounted for by the use of an eddy viscosity model. As can be seen a non-uniform eddy viscosity coefficient K is to be used. Outside the jet flow K is set equal to zero so that the governing equation is essentially the acoustic wave equation.

(3) Radiation and boundary conditions

Before the problem can be solved numerically boundary conditions must be prescribed. For the problem at hand the appropriate conditions are:

(a) On the jet axis, i.e. at $r = 0$

$v = 0$ for all z

(b) At the nozzle exit the upstream disturbances are to be simulated by using a random number generator subroutine. The spectrum probably resembles that of white noise.

(c) On the remaining part of the boundary an outgoing wave condition is to be imposed.

(4) Coordinate transformation

A jet flow is highly nonuniform with very large velocity gradient in the mixing layers. In order to properly account for the rapid change in velocity that exist in these regions a higher resolution is needed there in the computing scheme. This is achieved by using a coordinate transformation. The following transformation from the physical plane (γ, z) to the computing plane (ξ, η) will be used.

$$\eta(\gamma, z) = \begin{cases} 1.25 + \frac{1}{1.24}\tan^{-1}[4\tan 1.24(\frac{\gamma}{2b} - 0.5)] \\ \quad ; \text{ if } 0.25 < \frac{\gamma}{2b} < 0.75 \\ \frac{\gamma}{2b} \quad , \text{ otherwise} \end{cases}$$

$$\xi(z) = \begin{cases} 11.0 + \frac{1}{0.155}\tan^{-1}[0.5\tan 1.24(z - 5.0)] \\ \quad ; \quad 3.0 < z < 7.0 \\ z \quad ; \text{ otherwise} \end{cases}$$

The numerical coefficients of the transformation were selected after a few trials. Figure (1) shows a computer graphic of the grid points in the physical plane that are mapped into uniform rectangular meshes in the (ξ, η) plane where computation is to be performed. Notice in figure (1) very high resolution is attained in the mixing region of the jet and around the neighborhood of $z = 5D$. Figure (2) is an enlarged map of the part of figure (1) which is close to the nozzle exit (located at $z = 0$).

(5) Finite difference equation

To solve the governing equation numerically it is necessary to convert it into a finite difference equation. After a good deal of consideration and experimentation the two step Lax-Wendroff-Richtmyer method is adopted for this purpose. Without going into fine details the form of the difference equation is:

Step 1

$$u_{ij}^{k+1} = \frac{1}{4}\left(u_{i,j-1}^{k} + u_{i,j+1}^{k} + u_{i-1,j}^{k} + u_{i+1,j}^{k}\right) - \Delta t\left(E_{ij}^{k}\right)$$

$$E_{ij}^{k} = -H_{ij}^{k} + f_{ij}\,\frac{F_{i+1,j}^{k} - F_{i-1,j}^{k}}{2\Delta\eta} + g_{ij}\,\frac{G_{i+1,j}^{k} - G_{i-1,j}^{k}}{2\Delta\eta}$$

$$+ h_{ij}\,\frac{G_{i,j+1}^{k} - G_{i,j-1}^{k}}{2\Delta\xi}$$

Step 2

$$u_{ij}^{k+2} = u_{ij}^{k} - 2\Delta t\left(E_{ij}^{k+1}\right)$$

E_{ij}^{k+1} is the same as in step 1 with $k+1$ substituting for k

(6) Present status of this project

At the present time several subroutines relating to the coordinate transformation and mean flow calculation have been developed and stored in permanent file at the FSU Computer Center. Programming of the Lax-Wendroff-Richtmyer scheme is currently underway. It is expected that this time consuming step would be completed in a few months time. This includes debugging and the incorporation of boundary and radiation conditions. Some preliminary results should be available within five to six months.

Summary of Grant Support and Publication

Total funding of grant NASA No. NSG-1021, June 1, 1974 to June 30, 1976 —— $29,023

[redacted]

Support given to faculty — one summer (approximately)

Support given to graduate research assistant — $1\frac{3}{4}$ years (approximately)

Publications (Dr. Tam):

Trailing Edge Noise (with J. C. Yu) AIAA Paper 75-489, 1975.

The Transmission and Reflection of Sound in Curved Bends of a Duct System, Proceedings of the Third Interagency Symposium on University Research in Transportation Noise. Nov. 1975, PP. 511-521.

A Study of Sound Transmission in Curved Duct Bends by the Galerkin Method, Journal of Sound and Vibration, 45, 91-104, 1976.

Work in Progress:

"Numerical Experiments on a Mechanism (Large Scale Disturbances) of Jet Noise Generation" — Doctoral Thesis.

Figure (1)

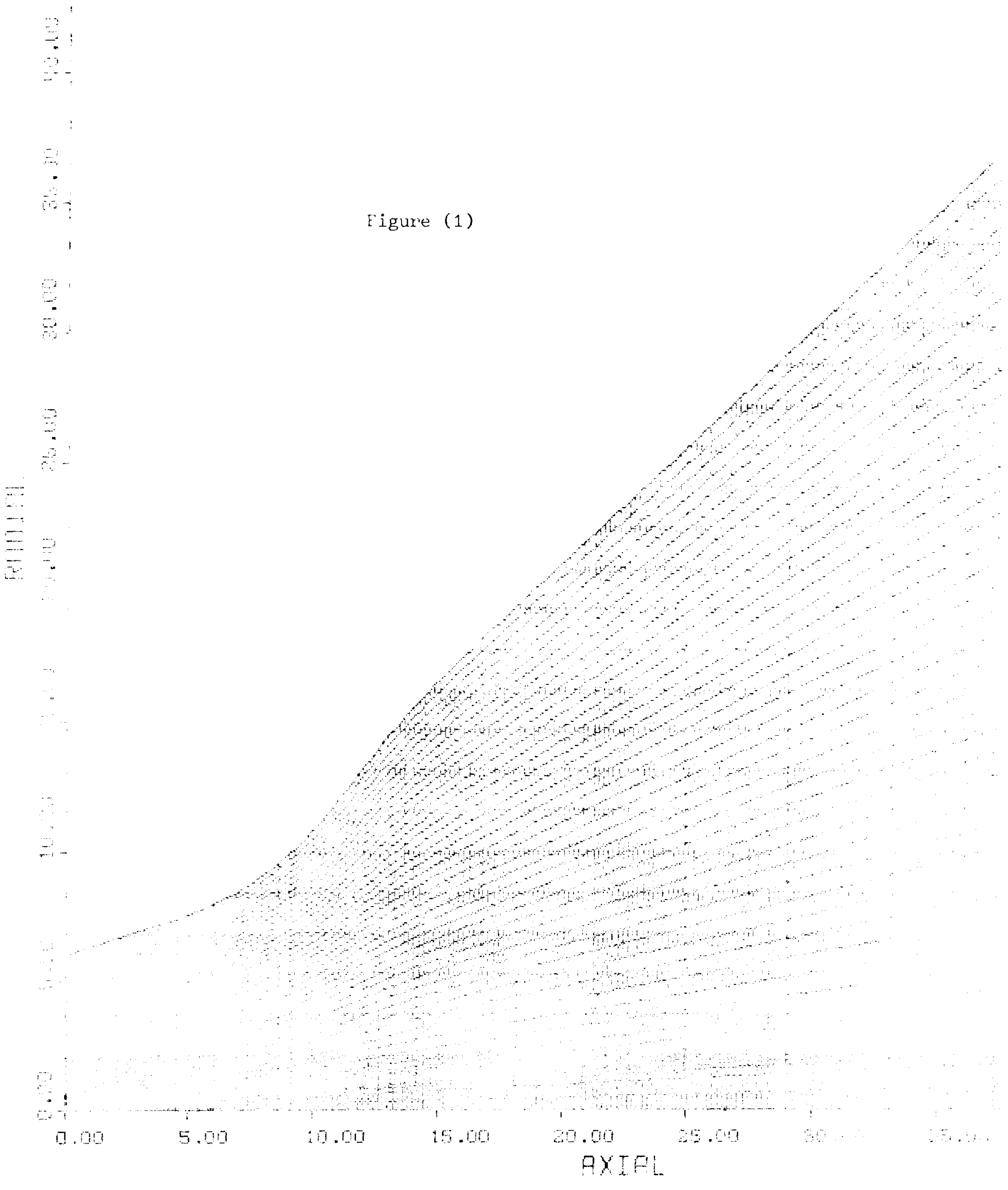

Figure (2)